Origins of the logical theory of probability: von Kries, Wittgenstein, Waismann[1]

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The physiologist and neo-Kantian philosopher Johannes von Kries (1853-1928) wrote one of the most philosophically important works on the foundation of probability after P. S. Laplace and before the First World War, his *Principien der Wahrscheinlichkeitsrechnung* (1886, repr. 1927). In this book, von Kries developed a highly original interpretation of probability, which maintains it to be both logical and objectively physical. After presenting his approach I shall pursue the influence it had on Ludwig Wittgenstein and Friedrich Waismann.[2] It seems that von Kries's approach had more potential than recognized in his time and that putting Waismann's and Wittgenstein's early work in a von Kries perspective is able to shed light on the notion of an elementary proposition.

1. Von Kries's logico-objective approach to probability

As a neo-Kantian, von Kries is very much opposed to any form of psychologism that he saw as prevalent in the usual conception of probability of his time. In order to eliminate any subjective elements from the notion of probability, he sets out to develop a physically objectivist account as a basis for a logical interpretation. He argues that probability must be based in some way or other on physical, objective features of reality and not on some degree of subjective certainty. The required foundation is to be introduced by way of the concept of "objective or physical possibility" (Kries 1886, 87; cp. 1888, 180-2). At first glance, this seems to be a self-contradictory procedure, because the term "possibility" appears to admit only of a subjective sense, relative to our state of knowledge. To say that something is "possible" seems to imply that we are "uncertain" of it under the known circumstances.

In order to make room for the idea of an objective possibility, von Kries introduces a distinction which is to play an important role not only in his probability theory but in his whole conception of logic, causality and science. It is the distinction between "nomological" and "ontological" determinations of reality and the two different kinds of empirical knowledge arising with them (Kries 1886, 85-89, 99, 172; 1888, 181f., 189; 1916, 53f. & Ch. 23; 1925, 158f.; 1927, x-xi.). Nomological claims characterize classes of things and are expressed as laws of nature, whereas ontological claims refer to contingent individual features of singular events, or to actually obtaining singular boundary conditions. In von Kries's words, "the former refers to the totality of laws expressed in reality, the latter to the purely factual modes of behaviour that are not determined by these laws" (1916, 53). Ontological claims “contain the purely factual; what cannot be reduced to any general necessity” (1886, 86). It turns out that the two kinds of knowledge are independent of each other: if we lack in knowledge of an ontological feature of the world, we would not be able to acquire it from whatever nomological knowledge we might happen to have; and if we are ignorant of a law of nature, finite ontological information does not do much to help us out of our state of ignorance.

Taking this distinction between nomological and ontological determinations into account, von Kries argues, we can talk of objective possibilities in a scientifically respectable and significant way. To say that an event is objectively possible means that it is compatible with a certain natural law or a set of laws, whatever its special ontological qualification. Its ontological determination is left open by the nomological setting, or, to put it differently, the event in question is not excluded solely by the natural laws under consideration. Any sentence stating an objective possibility of an event thus expresses knowledge of a nomological kind; it gives a set of general conditions or constraints. Yet at the same time, it leaves open the special ontological features of the world that lead in the end to the occurrence or non-occurrence of a specific event.

If we analyse this further, we can say that, relative to certain laws of nature L, it depends on a particular range of ontological conditions E_1 or E_2 or ... or E_n, whether a certain state of affairs H obtains or not, other things being whatever they may (my notation.). The fulfilment of each of these conditions is an objective possibility. Take for example the throw of a die. The laws of mechanics leave open the special ontological determinations of the throw, its spatial position, velocity and direction. Each of these conditions is causally relevant and will make the throwing of the die result in a certain outcome. The nomological set-up itself leaves a certain range of ontological conditions indeterminate or undecided (*unbestimmt*) where each condition, if it were realized, would lead to a certain definite outcome.

Von Kries calls a range of objective possibilities of a hypothesis or event (under given laws) its *Spielraum* (literally: play space), which can mean ‘room to move’, ‘leeway’, ‘latitude of choice’, ‘degree of freedom’ or ‘free play’ and ‘clearance’ - or even ‘scope’. John Maynard Keynes translated it as ‘field’, but the term ‘range’ has generally been adopted in English. Von Kries now holds that if numerical probability were to make any sense at all it must be through this concept of the *Spielraum*. Von Kries’s theory is therefore called a ‘*Spielraum* theory’ or ‘range theory of probability’.

The basic idea is that probability expresses the proportion in which a certain event occurs in the range of all those conditions that are nomologically relevant for its occurrence or non-occurrence. In general terms this would mean the following: take the disjunctive set E of ontological conditions E_1 or E_2 or ... or E_n which constitute the whole *Spielraum* of a state of affairs H relative to certain laws L. Together with this set of laws some of these alternatives imply the occurrence of H, and the remaining ones imply the occurrence of not-H. In this way, the *Spielraum* E is divided into two parts. (Call them E_H and $E_{\neg H}$ respectively.) If we call the first part of alternatives *favourable* to H, we can say that the probability of H relative to E and L is the ratio of the number of favourable alternatives of the *Spielraum* to the number of all alternatives, i.e. the whole *Spielraum*. “The size of the possibility which a condition has for an effect can be designated by the ratio of the *Spielraum* bringing about the effect to the entire *Spielraum*” (Kries 1888, 184, fn. 2).

In order to facilitate the discussion in the 2nd part of this paper, I shall try to express this formally, although von Kries never did this himself. He could have written this in the following way:

$$(1) \qquad \text{prob}\,(H/E) = \frac{\mu\,(E_H)}{\mu\,(E_H \vee E_{\neg H})}\,.$$

(Note that the formula in the parentheses of the denominator is not a tautology but the set of all state descriptions that are possible under L!)

The major difficulty now is to find a numerical and objective measure μ of the ranges. Von Kries is very much concerned to discover a way in which the number of alternatives can be determined without playing again the old Laplacean game of the Principle of Insufficient Reason and thus falling back into what he takes as subjectivism and arbitrariness. The solution with which he comes up clearly stands in the tradition of Jacques Bernoulli and Leibniz who tried to find a *physical* attribute as such a measure and who believed equipossibility to be the key to measuring probability. (See Kries 1886, 269-272 for von Kries’s discussion of Bernoulli; cp. Hacking 1971b, 345 and Hacking 1971a for the tradition of equipossibility.)

Von Kries tries to solve his problem from two angles: from a *theoretical* one, by investigating the conditions under which two *Spielräume* are equally possible, and from a *practical* one, by determining the objective features of games of chance (*Zufalls-Spiele)* that can serve as a foundation for the numerical expression of probability and thus for its mathematical theory. As a result of the first approach, von Kries arrives at his so-called *Spielraum* principle: a hypothesis has a certain determinate, objective, meaningful and numerically determinate probability if and only if, as von Kries says, its *Spielraum* is indifferent, original and comparable (*indifferent*, *ursprünglich* and *vergleichbar*) (1886, 36f.).

These three conditions are defined in the following way: the *Spielraum* of a hypothesis is *indifferent* if there is conclusive reason to believe that it can be subdivided into a set of exclusive and exhaustive equal alternatives such that none of them is more favourable to the outcome than any other. So, if we know for example that a die’s centre of gravity is not eccentric etc. we are justified in believing that, whatever the initial conditions, none of the results is favoured over another. We arrive at six equal alternatives corresponding to the six outcomes of throwing the die. Von Kries thus abandoned the reliance on the Laplacean “Principle of Insufficient Reason” in favour of, as he calls it, the “Indifference Principle”. (Unfortunately, Keynes, who was much influenced by von Kries, used this expression as a new terminology for the Laplacean principle and thus obscured and thwarted von Kries’s own discussion and clarification. See Keynes 1921, 41, 88.)

Second, a *Spielraum* is said to be *original* or ultimate, if the sizes of the alternatives remain stable or stationary when taking into account the prehistory of the individual alternatives; or, to put it differently, if a *Spielraum* cannot be derived from another, more fundamental one. In this case, going back to earlier conditions of the event in question would not change the picture in any way. If, for example, the throw of a die were manipulated or the die itself loaded, the *Spielraum* resulting from taking each of the six sides as equal would not be original, because going back in the history of throwing the die would show that one or more sides were favoured over the others. Originality frees us from being dependent on limited information of the situation.

And finally, a *Spielraum* is *comparable*, if there is a unique, objective, non-arbitrary and compelling method of subdividing it into a set of mutually exclusive equal units. In the simplest case it would have to be guaranteed that the possibilities are of equal size. This is the most crucial condition.

Before discussing how von Kries solves the problem of partitioning a *Spielraum* into equal units, let us ask why these three conditions secure the numerical character of a probability, as von Kries claims. They allow us to objectively define equality of range: two ranges are equal if they are both indifferent and original and if they have an equal amount of comparable units in common. This in turn allows us to define what it is for two assumptions to be equally probable. As soon, however, as it is objectively meaningful to speak of equality in probability, it makes sense to express probability by a number. As in the physical measurement of a length a numerical statement gets its meaning by objectively comparing the length with a unit measuring rod, so a probabilistic statement gets its meaning by an objective comparison of ranges. It follows that probability is not based on subjective uncertainty and thus has nothing to do with psychology, but can be founded on physical and objective features of the world. Mathematical probability theory can thus, at least in principle, be interpreted in an objective way.

At first glance, the reader might surmise that by arguing in this way von Kries will be forced sooner or later to rely again on Laplace’s Principle of Insufficient Reason. But note that for Laplace probabilities are based on a *lack* of knowledge and therefore, as one could say, on *negative* reasons, whereas von Kries’s intention is to identify all the *positive* and objective reasons that are necessary and sufficient for expressing probability in numerical terms. To vary an apt saying by William Kneale: whereas Laplace and his followers accept "absence of knowledge" as sufficient ground for meaningful judgements of probability, von Kries looks for "knowledge of absence" of grounds (Kneale 1963, 173. Cf. in a similar way Meinong 1890, 70.). He is thus enabled to sharply criticize Laplace's Principle of Insufficient Reason and to develop what Henri Poincaré later called Bertrand's paradox. (Kries 1886, 8f. Bertrand himself published his paradox two years after von Kries.)

It is true that all this does not say anything yet about the empirical practicability and meaningfulness of von Kries's approach. Everything depends on the way that the equality of ranges is to be determined empirically. It seems to be a hopeless enterprise to arrive at actual numerical probabilities of interesting empirical phenomena, except for the most simple and trivial case of symmetrical games. Von Kries is very well aware of this objection. He points out in his answer that there is at least one class of cases where we have the required definite knowledge of absence of grounds: the ideal games of chance. A probabilistic statement is all the more objective the more the underlying mechanism referred to resembles an ideal game of chance. This idea does not solve the problem in a straightforward way, but it is a definite advance over von Kries's predecessors.

In order to show that for an ideal game of chance the conditions of indifference, originality and comparability are definitely and objectively fulfilled, von Kries considers a simplified, straightened out roulette, or *Stoss-Spiel*, as he calls it, where a ball is pushed in a horizontal groove divided in infinitesimally small black and white stripes. Such a game is to be called a game of chance not because of our ignorance of the conditions nor because of an alleged suspension of a *nomological* necessity in nature, but because of the *ontological* arrangement of the situation: the infinitesimal variation of the initial conditions secures the periodical change of the outcome (Kries 1888, 187).

The probability that the ball stops on white or black is equal and depends only on the intensity of the push, i.e. the initial state. After the push, the behaviour of the ball is continuously variable, i.e. it does not make any leaps, and it is not periodic in a way that would favour black over white or vice versa. Because of the infinitesimal lay-out of the game, the probability of a certain position of the groove to be hit by the ball is equal to the probability of its neighbouring position. Each differential can thus serve as a unit of comparison for different parts of the *Spielraum*. In addition, we can be sure that no increase in nomological knowledge whatsoever would change our view of this situation, because it depends only on the knowledge of the initial conditions.

Yet neither can this kind of ontological knowledge ever be acquired, neither *ex ante* nor *ex post*, as von Kries says, because we will always commit an error in determining the ontological elements, and any such error, however small, will be too sensitive for the result. We could say, then, that an ideal game of chance is epistemologically robust or, as von Kries puts it, that its probabilities are *universally valid*, because they do not change in the light of whatever further (ontological) evidence with which we might become familiar (see esp. Kries 1888, 186 & 188f.; also 1886, 94f.). All this shows that the *Spielraum* of an ideal game of chance such as the *Stoss-Spiel* fulfils the theoretically stipulated criteria, i.e. that it is objectively indifferent, comparable and original. It therefore can be divided into equal parts of objective possibility and thus can serve as an objective basis for a numerical theory of probability. "Numerical probability represents an *ideal case of logical behaviour*, a case, which is realized with utmost approximation, but never with absolute precision [in an ideal game of chance]" (1886, 77f.). The equiprobability of an ideal chance set-up is thus reduced to the existence of a continuous probability function.

The restriction of objective numerical probability to ideal games of chance greatly diminishes their applicability to empirical cases. In a strict sense, meaningful numerical values for empirical phenomena can only be obtained if it is certain that they approximate sufficiently an ideal game of chance. To test this, von Kries recommends the method of dispersion, as developed by the statistician and economist Wilhelm Lexis in 1876, which measures the independence of the results from each other (Kries 1886, 144-53, 104-09). Von Kries sees also the possibility to *estimate* the probability of an empirical event by comparing its experienced relative frequency to a certain objective numerical probability. Such an estimation, however, is of a psychological nature: it says that the empirical event is to be expected with the same certainty as an outcome of a certain corresponding ideal chance set-up (Kries 1886, 181-85).

It is of special interest that von Kries included a chapter in his book on the application of probability theory to Ludwig Boltzmann's kinetic theory of gases (Kries 1886, Ch. VIII). He maintains that if Boltzmann's findings were reformulated in an appropriate way one could show that the ranges involved are indifferent and comparable, and most importantly, also original. This proves for him that the Second Law of Thermodynamics, although stating a regularity, is rightly to be understood as a probabilistic law and not as a nomological necessity.

In order to show this von Kries argues in the following way: from the *Spielraum* Principle it follows that the probability of a certain state is always equal to the probability of a later state for which it is a necessary and sufficient condition. This formulation might seem to be in conflict with the Second Law, which says that the less probable state regularly changes into a more probable one. This, however, is tantamount to attributing the highest probability to those *original* ranges (in von Kries's sense) which change into a state of equilibrium. If the physicists call this state of equilibrium "the most probable state" it should be kept in mind according to von Kries that this expression does not refer to a single special state (which would always be most improbable), but to a set of many extraordinarily different states. The ranges are comparable because infinitesimal variations in location and velocity of the molecules will lead to arbitrarily large differences in the outcome after a collision (Kries 1886, 209f.). It follows for von Kries that probability can be applied in the case of the theory of gases as in all other cases because the behaviour follows from the slightest indeterminacy of the initial state, however small (Kries 1886, 58).

For a contemporary reader the puzzle might remain why von Kries calls his physical interpretation of probability also a "logical" one. This is because it states a necessary relation between the event in question and the *Spielraum* of the hypothesis. If we want to know the probability of an occurrence we ask to what degree its expectation is supported by our objective knowledge of the *Spielraum* in question. To say that the probability to throw a four is $1/6^{th}$ means to say that the statement "a four will be thrown" is justified to the degree of $1/6^{th}$ by the statement "to throw a one or a two or … or a six is equally possible". In this sense, the numerical value in a probability statement "can be conceived neither as a measure of an actual psychological state, nor as a consequence that results from practical principles, but as a *logical proportion*" (Kries 1886, 5f.). The past leaves for the future a certain unique physical *Spielraum* which is empirically given. If it is known you can assess logically to what degree a certain future event participates in it. This is its probability.

2. Waismann's elaboration of von Kries's theory

As far as we know, von Kries's interpretation of probability found its most sympathetic reception with Wittgenstein's disciple Friedrich Waismann. In his well-known article on "The logical analysis of the concept of probability" of 1930 Waismann uses the term "*Spielraum*" as a matter of course (Waismann 1930, 10 [235]; here translated as "scope"). Waismann does not refer to von Kries by name, but it is beyond question that von Kries's book was the starting place for his thoughts. Waismann gave his paper, as did Reichenbach, von Mises, P. Hertz and Feigl, in the philosophy of probability section of the *1. Tagung für Erkenntnislehre*, organized by the *Verein Ernst Mach* (Vienna) and the *Gesellschaft für empirische Philosophie* (Berlin), in Prague in September 1929. The proceedings of the congress were subsequently printed in the first volume of *Erkenntnis*. In the article following directly after Waismann's, Feigl commented that Waismann's "definition of the concept of probability expresses exactly what von Kries meant by 'ranges of possibilities' [*Spielraumverhältnisse*] and all that which one traditionally wanted to express by the really imprecise concept of 'objective possibility' " (Feigl 1930, 107 [249]).[3]

Waismann starts his paper with a generalisation of the *Spielraum* conception. (In a footnote he asks the reader to compare this with Wittgenstein's *Tractatus* 5.12ff.[4]) A proposition, he says, does not just lay down a fact, it establishes a *Spielraum*, a range of facts. As long as the facts stay inside its *Spielraum* a proposition is true. It is false as soon as its *Spielraum* is transcended by the facts. He then goes on to show the relevance of the *Spielraum* conception for the notion of logical consequence. A sentence follows from another one, if its *Spielraum* encloses the one of the first. If the two *Spielräume* do not overlap, the corresponding propositions contradict each other. "Entailment and contradiction are represented in this picture as – as it were – topological relations between scopes [*Spielräume*]" (Waismann 1930, 10 [236]). After that he develops the concept of a measure of the size of a proposition's *Spielraum* μ (p) in the following way:

(i) $0 \leq \mu(p) \leq 1$,

(ii) a contradiction has the measure zero,

(iii) if two propositions p and q are mutually exclusive, then $\mu(p \vee q) = \mu(p) + \mu(q)$.

He then gives his definition of the probability a proposition q gives to a proposition p: it is the size of the *Spielraum* of p and q together, in relation to the size of the *Spielraum* of q alone, or:

(2) $$\text{prob}(p/q) = \frac{\mu(p \,\&\, q)}{\mu(q)}.$$

(See Waismann 1930, 11 [237]; notation slightly adjusted.) As Waismann notes, this definition of probability gives a measure of logical proximity existing between two propositions.

In the course of his argument, Waismann identifies a deep-seated analogy of probability theory with geometry. He observes that "the propositions of geometry do not describe our actual measurements, they are the rules according to which we interpret those measurements." And he continues that

> just as the laying-down of the axioms of geometry is determined by the consideration that their choice lead to laws of nature of the greatest possible simplicity, so the choice of a metric in the probability calculus is guided by considerations of utility. (Waismann 1930, 17 [244]).

In a sense then, Waismann does to von Kries in probability theory what Poincaré once did to Hermann Helmholtz (and Kant) in geometry. In Helmholtz's view, the axioms of geometry do not only say something about space, but also about the mechanical behaviour of rigid bodies during motion (Helmholtz 1870, 244 [29]).

In an analogous way, Waismann maintains that the choice of the metric for the *Spielraum* rests on a convention and can be chosen freely. It is guided by considerations of simplicity in accounting for the experienced frequencies. He illustrates his view by a discussion of the Galtonian board (the quincunx). We do not know whether a ball goes to the left or to the right after it has hit a pin. If we conventionally lay down that the two possibilities have the same probability we have thereby fixed the metric which is the origin of all further mathematical considerations. The Gaussian bell-shaped curve is thus "the ideal according to which the degree of approach of the actual distributions is judged." We say that it is a 'matter of chance' whether the ball goes to the left or to the right after hitting the pin because we think that this event is independent of all other circumstances. Therefore, "to give the measure of probability is thus to stipulate when we shall speak of chance and when not" (Waismann 1930, 18 [246]). Whereas von Kries had taken the behaviour of an ideal game of chance as a 'rigid rod', so to speak, with which all other probabilistic phenomena are to be compared (as little as this might be achievable according to his original account), Waismann takes the ideal game of chance as a *convention* which has to be fixed before probability theory can take off.[5]

It could very well be that Waismann was inspired in his view by Poincaré himself who wrote in 1902 that "to undertake the calculation of any probability, and even for that calculation to have any meaning at all, we must admit, as a point of departure, an hypothesis or convention." He then continued, however, that in the choice of this convention "we can be guided only by the principle of sufficient reason" and by our belief that the probability function, such as the one describing the outcome of a roulette, is continuous (Poincaré 1902, 210; cp. also 201). This still subjective view is corrected, however, six years later, when Poincaré points out that "chance, then, must be something more than the name we give to our ignorance." Similarly to von Kries, he takes chance as instability of the initial conditions of a game of chance (Poincaré 1908, 66; Plato 1983, 38f.; Plato 1994, 170).

Waismann continued his discussion with a searching critique of the empirical interpretation of probability – a critique that again has much in common with von Kries's. He rejects the frequency interpretation for resting on a completely misconceived conception of idealisation in science. (Waismann acknowledges for the following his debt to Wittgenstein: see 1930, 8, 21 [233].) If we measure the circumference and radius of different circles and we find values which do not agree with the number π do we say that geometry has been refuted? No. But neither do we take π as an ideal limit which we learn to know better and better through our experiences in measuring circles. On the contrary, π is the measure which allows us to judge the measurement of a circle as successful. "To idealise does not mean: to mentally conceive of empirical measurements as refined beyond limit, but it means to describe the observed phenomena with concepts of preconceived syntax." In the same way, Waismann holds, we should look upon the idealization which is built into probability theory. He criticizes von Mises for completely misjudging the role of idealization in taking an *empirical* sequence as a mathematical series and an *observed* relative frequency as an ideal infinite limit (Waismann 1930, 9 [234]; for von Mises see Plato 1994, Ch. 6 and Heidelberger 1987).

3. Wittgenstein's dependence on von Kries

As is well-known and universally accepted, Waismann's approach is an elaboration of ideas originally formulated in Wittgenstein's *Tractatus*. This makes it highly probable that Wittgenstein was also the source of Waismann's awareness of von Kries and that he (Wittgenstein) directly depended on von Kries in his conception of probability. (Von Wright 1982, 147 also deems it possible that Wittgenstein was familiar with von Kries.). It is highly significant that Wittgenstein's contemporary admirer Moritz Schlick put Wittgenstein in the same camp with von Kries in his praise of the *Spielraum* theory. After criticizing von Mises's definition of probability for relating to empiristically problematic infinite reference-classes, he wrote that the "only usable method for defining probabilities is, in fact, that which utilizes logical ranges [*logische Spielräume*] (Bolzano, von Kries, Wittgenstein, Waismann; see the latter's essay cited above [citing Waismann 1930])" (Schlick 1931, 201; see below for Bolzano).

Wittgenstein uses the term *Spielraum* twice in the *Tractatus*; in 4.463 and 5.5262. Although he does not apply it explicitly in connection with probability and never mentions von Kries by name, it is clear that he uses *Spielraum* in a sense that is closely related to von Kries's. In 5.101 Wittgenstein had defined "truth-grounds of a proposition" as "those truth-possibilities of its truth argument that make it true." This is followed by a definition of logical consequence that is similar to Waismann's. In 5.15 then, Wittgenstein defines probability in a way which is closer to von Kries's original formulation than to Waismann's:

> If T_p is the number of truth-grounds of a proposition 'p', and if T_{pq} is the number of truth-grounds of a proposition 'q' that are at the same time truth-grounds of 'p', then we call the ratio $T_{pq} : T_q$ the degree of *probability* that the proposition 'q' gives to the proposition 'p'. (notation slightly adjusted.)

There is, however, a crucial change in comparison to von Kries. Whereas von Kries takes the concept of the range of an event or proposition as entailing information about the actual laws of nature and therefore about the *empirically* possible world, Wittgenstein relates it solely to *logically* possible worlds. This move does not, however, constitute just a simple denial of von Kries's conception. It can and must be reconstructed as an answer to the following question: what would be the (least damaging) consequences for von Kries's general outlook if probability were to relate solely to logical and not to physical possibility? The only consistent answer would be that we have to deny that reality is nomologically determined at all. The *Tractatus* abounds, of course, with statements that embrace this consequence wholeheartedly (in the strongest way to my mind in 5.135, 5.136, 5.1361). The rejection of the nomological dimension of the world forces us to limit ourselves to determining its *ontological* arrangement in our cognitive enterprise.

The rejection of physical possibility as a basis for probability and its restriction to logic does not, however, give us a new method of measuring the size of a *Spielraum*. We have to retain von Kries's idea of the *originality* of a *Spielraum* and thereby build our ontological description of the world on the disjunction of alternative conditions that are original, i.e. independent of each other. This immediately yields Wittgenstein's concept of an elementary proposition: take the disjunctive set of logical possibilities in and for our world that forms an original, indifferent and comparable logical *Spielraum*. A proposition is elementary if it corresponds to a member of this set in a non-negated form.

The forgoing shows that interpreting the *Tractatus* from the perspective of von Kries can help to explain where the concept of an elementary proposition in Wittgenstein comes from and what it exactly means. This reading can also make clear why two elementary propositions have to give one another the probability ½, as Wittgenstein maintains in 5.152, and why an elementary proposition itself does not give a *Spielraum* to reality, as non-elementary propositions do.

McGuinness denies that Wittgenstein was familiar with von Kries because he did not use the notion of *Spielraum* in connection with probability. But he does use it in connection with the truth-conditions of a proposition (4.463 and 5.5262) – a conception, as we have seen, that is the result of extending the consideration of 'objective possibility' beyond probability statements to propositions in general (McGuinness 1982, 165; 1989, 185. Cp. Black 1964, 234f., 247-258.). I see Wittgenstein's idea of truth-function and logical space as a direct outcome of generalizing the notion of objective possibility and at the same time restricting it to logic and propositions.

We still have to come back to Schlick making Bernard Bolzano a proponent of the range theory of probability. (For Bolzano's concept of probability see Schramm 1989 and Reitzig 1973, xxi-xiv.) Although Wittgenstein's ideas on probability do have a certain similarity with Bolzano's of 1837, we cannot infer that Wittgenstein was influenced by them until there is more independent evidence. The present temptation to stress Wittgenstein's Austrian roots at the expense of any neo-Kantian (and other) influences should not too hastily be followed. I would, however, not go as far as von Wright and reject that Wittgenstein ever heard of Bolzano's notion of probability (Wright 1982, 145). This might be true for the period before 1930 but not for the later one. Taking the discussions on probability into account Wittgenstein had with Schlick and Waismann in January and March 1930 (Wittgenstein 1979, 93-96, 98-99), it is likely that Wittgenstein did read Waismann's paper at some point at that time. He must have then come across its reference to Bolzano (Waismann 1930, 228) and could have noticed a few pages further in the same issue of *Erkenntnis* the minutes of the Prague discussion on probability, where Dubislav explicitly compared Bolzano's and Waismann's ideas (Zilsel et al. 1930, 264-266).

There could, however, very well have been an Austrian after all who put Wittgenstein on the von Kries track. It was Boltzmann who, in the same year in which von Kries's book appeared, praised it for containing a "logical justification" of the numerical calculation of probability (Boltzmann 1886, 242). It is difficult to imagine that such a devoted admirer of Boltzmann as Wittgenstein was (Wittgenstein 1980, 19) with such an intense interest in logic should have missed this remark. We know that Wittgenstein bought Boltzmann's *Popular Writings* containing a reprint of the address with the reference to von Kries (Boltzmann 1905, 37) soon after their appearance in 1905 (Wilson 1989, 257).

4. Conclusion

The von Kries perspective does not only help to clarify Wittgenstein's notion of probability but can also give us a clue in which general direction Wittgenstein headed with his *Tractatus* compared with von Kries's own account. It seems to me that it strives to strengthen, and in a way to complete, the anti-psychologistic and anti-naturalistic neo-Kantian tendency of its predecessor. Not only the relation of probability to experience, but the relation of language to the world in general, can be explained solely by reference to the logical form and to the ontological structure of the world. In order to close any possible loophole for a different account, the existence of nomological necessity internal to the world is rejected. It would therefore be completely wrong to see elementary propositions as an empirical fundament from which we can begin a logical construction of the world, but as the condition of the possibility of meaningful language as such.

However, with Wittgenstein's and Waismann's amputation of the physical component of von Kries's theory and with the subsequent reformulation of its logical remains in terms of expressions in an artificial language by Rudolf Carnap, a challenge has been missed and a great opportunity lost that could perhaps have been a genuine empiricist alternative to the relative frequency empirical interpretation of probability. Von Kries's account of probability could have become a paradigmatic case of how logic *and* experience are amalgamated in our epistemological concepts. Instead, the Logical Empiricists stuck to the frequency interpretation or unimaginatively split probability up into 'probability$_1$' and 'probability$_2$' (Carnap). The task would have been to find something like a 'correspondence principle' for probability, understood as a theoretical term, which arises from the special lay-out of the ranges, or their ontology, as von Kries would have said. This task would certainly not have been trivial, and there is, of course, no guarantee for its success. Sighed Hasso Härlen in the Prague discussion of 1929: "The abyss between theory and experience comes fully to the fore only with probability theory" (Zilsel et al. 1930, 267).

Notes

[1] A preliminary version of this paper was presented at the 2nd International History of Philosophy of Science Conference at the University of Notre Dame, IN in 1998. Additional research for it was conducted during my time as fellow at the Center for Philosophy of Science of the University of Pittsburgh in 1998-1999.

[2] I shall build in part on previous accounts by Kamlah and von Plato: Kamlah 1983; 1987a, 109-111; 1987b, 316-20; 1989, 436-443; Plato 1983, 38f.; 1994, 169f. The rich 19th-century context of von Kries's ideas, as well as the varied and profuse reception of his theory beyond Wittgenstein and Waismann will be treated separately elsewhere.

[3] Here and in the following page numbers in brackets refer to the original German edition.

[4] Here and in the following see Wittgenstein (1921, 1989) for reference to Wittgenstein's *Tractatus*.

[5] It should be noted that in his review of von Kries's book, the above mentioned statistician Lexis had already maintained that a game of chance rests on the *agreement* between the players to regard certain cases as equipossible (Lexis 1886, 435f.).

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