

## On Identity Criteria

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### Introduction

Consider this example. We bring a friend into a room. There is a table, a chair, a book, and a cat sleeping on the chair. Nothing else. We ask: “How many objects are there?”. Suppose that the friend’s answer is: “Four”. Again, we ask: “Which objects are there?”. Answer: “A table, a chair, a book, and a cat”. We insist: “Do you not count yourself?”. Answer: “Then, five”. “And what about the pages of the book? The chair legs? The cat’s tail?”. And so on (the example is taken from [Putnam 1988, cap. 7]). One way to stop this chain of queries is to argue that the first one does not hold a determinate meaning. It would be rightly raised if there was a specification of the kind or sort of objects to count (this solution is advanced, among the others, – with some differences – by [Dummett, 1981, p. 547]; [Geach, 1980, p. 63]; [Wright, 1983, p. 3]). Therefore, according to this solution, it is appropriate, where the use of “appropriate” is the common one, to ask how many tables are in the room, or how many chairs, or how many books, or how many cats, or how many human beings... because in all these cases there is a suitable specification of the query at issue. Doing so, we associate to the term for a *kind* or sort — the *sortal term* — in some way, an *identity criterion*. So, a question concerning counting is determinate, or appropriate, if and only if it is specified by a sortal and there is a *criterion of identity* for the kind (or sort) of the term.

The goal of this paper is to analyse the expression “criterion of identity” occurring in the above biconditional. We want to *exclude* that some *standard* meanings are the *meaning* of the expression discussed. If criteria of identity are used in that way, in order to specify a question concerning counting, and if the specification of the question is the solution proposed to stop the chain of queries produced by the first question: “How many objects are in the room?” then the solution is false.

We will move on in the following way. We will start with a Fregean quotation of the *Grundlagen* where the expression “criterion of identity” has been introduced. The analysis of the quotation will allow us to distinguish two meanings, expressing two functions, usually ascribed to identity criteria. Roughly, the first one provides the ontological conditions for identity. It expresses an ontological or metaphysical function of identity criteria (the two terms are here equivalent). The second one provides the epistemic conditions for the identity knowledge. It expresses an epistemic function of identity criteria. We assume that the *standard* meaning of the expression

“identity criterion” is given either by the *ontological reading* or by the *epistemic one*.

We argue that it is false to say that identity criteria are epistemic conditions for the identity knowledge, and that it is false, under some conditions – and we stress *just under some conditions*, where the conditions will be explicated in the paper – to say that identity criteria are ontological conditions for the identity.

If a question concerning counting is appropriate if and only if it is specified by a sortal term and by an *identity criterion* for the kind of the term, and if the term “identity criterion” occurring in the second part of the conjunction is equivalent to “identity criterion in its *standard* meaning” then the solution is false.

## 1. A preliminary characterisation of the ontological and epistemological function of identity criteria

The credit for introducing the notion of identity criteria is attributed to Frege. In the *Grundlagen* §62 he wrote:

If we are to use symbol *a* to signify an object, we must have a criterion for deciding in all cases whether *b* is the same as *a*, even if it is not always in our power to apply this criterion [Frege 1884, §62].

The example of identity criterion, proposed by Frege, concerns directions of lines in terms of parallelism. He remarked:

The judgement ‘line *a* is parallel to line *b*’... can be taken as an identity. If we do this, we obtain the concept of a direction, and say: ‘the direction of line *a* is identical to the direction of line *b*’ [Frege 1884, §64].

*Prima facie*, the proposed example suggests that the question to be answered by an identity criterion can be stated in the following general way:

(OQ) If *a* and *b* are Ks, what is for the object *a* to be identical to *b*?

We call (OQ) the “ontological question”. Usually, an answer to this question should give the necessary and sufficient conditions for *a* to be identical to *b* under a sort K.

If we review the first passage quoted from Frege one could observe that the ontological reading is not the only one. From Frege we could take the suggestion that there is a second function of an identity criterion, and it is to answer the following question:

(EQ) If  $a$  and  $b$  are Ks, how can we know that  $a$  is the same as  $b$ ?

We call (EQ) the “epistemic question”. The identity criterion, in this perspective, concerns the knowledge of identity between entities  $a$  and  $b$  of a sort K. Usually, the epistemic function seems to be required when we talk of the identification of an object through time. Consider a typical question concerning identification through time like this one:

How can we *know* whether the table that is here now is the same as the table that was here yesterday?

This question is just an instance of (EQ) (on this issue see [Jubien 1996, pp. 345-346]).

## 2. The epistemic function of identity criteria

There is a reading of the epistemic function of identity criteria which immediately raises a problem.

If identity criteria are infallible recipes for working out the problem of recognition of an entity then there are none. In fact, it is impossible for a criterion of identity so understood to provide a general method for determining the truth of any identity statement concerning objects of a kind K. Let us suppose that a criterion of identity for the Ks provides a way of discovering whether *any* sentence of the form  $a = b$ , when  $a$  and  $b$  are Ks, is true or false. It would provide a way of discovering whether any sentence whatever is true or false. But this is absurd [Lombard 1986, p. 246].

Here is the argument. Take the identity statement:  $a = (\iota x)(x = a \wedge S)$ , where S is any sentence. It is logically true that:

S if and only if  $a = (\iota x)(x = a \wedge S)$

(Case left-right: S is true. Then  $(a = a \wedge S)$  is true. Hence there is an  $x$  such that  $(x = a \wedge S)$  and  $a$  is the only object satisfying the condition:  $(x = a \wedge S)$ , i.e.  $a = (\iota x)(x = a \wedge S)$ . Case right-left:  $a = (\iota x)(x = a \wedge S)$  is true. Then  $a$  satisfies the condition:  $(x = a \wedge S)$  and so S is true). It follows that, since a criterion providing a way of discovering whether an identity sentence is true or false will provide, in particular, a way of discovering whether an identity sentence of the form  $a = (\iota x)(x = a \wedge S)$  is true or false, the same criterion will provide a way of discovering whether any sentence whatever is true or false.

So we have to exclude this reading of the epistemic function. Identity criteria are *not* infallible recipes for working out the problem of recognition of an entity.

### 3. The ontological function of an identity criterion

The second reading of the first Fregean quotation is the ontological one. The question is (OQ) and its answer could be stated in this way:

(AOQ) Identity criteria explain what identity sentences mean.

A first way of interpreting (AOQ) is to take identity criteria as providing a definition of identity. But, is identity definable?

The condition – already considered by Leibniz and Frege as a possible definition of identity – is to share all the properties. More recently, the possibility of defining identity in this way has been considered by Brody in [Brody 1980, p. 8]. He defends it against this objection: reference to all the properties is circular because it involves, in particular, reference to properties presupposing the identity itself. According to Brody, such a criticism makes the definition impredicative, but not circular. Actually, there is no formal circularity in such a definition, but it seems to us difficult to disclaim that impredicativity is a kind of conceptual circularity. As concerns the identity definition at issue, it is at least not obvious that quantifying over all properties does not presuppose that a reference to identity is involved in (the formulation of) some properties.

The difficulty concerning the circularity of the definition of identity has a long story. It has been thought that it is possible to avoid it by narrowing the domain of quantification to properties not involving identity, for example to properties expressible in a language without the identity predicate. However, it has been observed that in this way a relation of indiscernibility is defined, which depends on what properties are quantified over. Being aware of this objection, Quine claims that the defined relation of indiscernibility may be taken as identity among suitable entities.

In general we might propound this maxim of the *identification of indiscernibles*: objects indistinguishable from one another within the terms of a given discourse should be construed as identical for that discourse. More accurately: the references to the original objects should be reconstructed for purposes of the discourse as referring to other and fewer objects, in such a way that indistinguishable originals give way each to the same new object [Quine 1953, p.71]. Wiggins' objection was that such entities are not, in general, those grasped by the language speakers. Moreover, speakers' intentions cannot be grasped by a suitable extension of the language, because the explication process could be endless. Wiggins criticises a possible, weaker, attitude, i.e. that: «In any given situation and any given context, all the identities of things involved are fixed or fastened down from some arbitrarily large sufficiency of information about all the various other predicates and relations that are instantiated there. ... Identity must *supervene* on other properties and relations» [Wiggins 2001, p.10]. Wiggins' criticism is

grounded on this thesis: issues concerning what is true of, i.e. about exemplification of properties and relations, presuppose that objects are already individuated, and therefore issues of identity are already settled.

We accept Wiggins' conclusion that identity is primitive and it is presupposed by the reference to objects. So, we deny that identity criteria, whatever they are, can give a definition or an explication of identity not presupposing the identity itself. This does not imply that identity criteria have no function at all. Frege himself assigns a relevant philosophical role to identity criteria, while maintaining that identity is primitive.

#### 4. Frege on the ontological function of identity criteria

Frege explicitly supports the priority of identity on identity criteria, even when identity criteria are suggested as explications of the sense of some specific identity sentences. In the passages above quoted from *Grundlagen* Frege seems to suggest that reference to objects presupposes the availability of identity criteria for the objects referred to.

He thinks of the identity criterion for numbers, as an explication of the sense of:

- 1) The number which belongs to the concept  $F$  is the same as that which belongs to the concept  $G$ .

To give an idea of what he is looking for, Frege considers the definition of:

- 2) The direction of line  $a$  is identical to the direction of line  $b$ ,

by:

- 3) line  $a$  is parallel to line  $b$  [Frege, 1884, §64, 65].

Frege observes that in (2):

The direction of  $a$  plays the part of an object, and our definition affords us a means of recognising this object as the same again, in case it should happen to crop up in some other guise, say as the direction of  $b$ . But this means does not provide us for all cases. ... That says nothing as to whether the proposition:

“the direction of line  $a$  is identical to  $q$ ”

should be affirmed or denied, except for the one case where  $q$  is given in the form of “the direction of  $b$ ”. [Frege, 1884, §64, 65].

In Frege's opinion, the nature of certain objects is entirely clarified only if one can find a way to refer to them in such a way that it allows us to decide the truth-value of any identity sentence concerning the given objects, and

not only those which that identity criteria state as equivalent. On the basis of the above quotation and the subsequent remark, we can reformulate (OQ) in this way:

(OQ1) If  $a$  is K, what is for the object  $a$  to be identical to  $b$ ?

A problem arising in reading Frege's attempt to exploit an identity criterion for Ks in order to get the concept of K concerns what is the sense in which he speaks of deciding identity sentences. We think that, for Frege, "to decide  $p$ ", where  $p$  is an identity sentence, means that the relevant information logically implies  $p$  or logically implies  $\neg p$ .

Of course, what information is relevant may depend on  $p$ , *a posteriori* information could be needed and, obviously, neither  $p$  nor  $\neg p$  should be included in it. Stating these requirements in a satisfactory way is difficult, perhaps, impossible. But this is not a Fregean problem. In fact Frege was surely worried only about certain abstract objects. For this kind of objects only general, *a priori* information, is relevant.

Moreover, it is very likely that Frege did not feel committed to providing a full explanation of the notion of identity criteria, since, very soon, he gave up the idea of exploiting this notion in order to introduce the concept of number.

Finally, let us focus on the fact that in his attempt to exploit an identity criterion for Ks in order to get the concept of K, Frege does not give up identity as a primitive notion. This comes out in the Fregean statement that the definition of identity among directions helps «to adapt the relation of identity, taken as already known, to a special case» [Frege 1884, §65], and in a preceding passage, where Frege argues that the goal of searching for an identity criterion for numbers is: «to use the concept of identity, taken as already known, as a means for arriving at that which is to be regarded as being identical» [Frege 1884, §65].

It is manifest that he takes the notion of identity as presupposed in the formulation of the identity criteria. This is not Quine's position.

## 5. Quine on the ontological function of identity criteria

Quine – as analysed in §3 – is prompted to consider identity as supervenient on some other properties and relations. The criterion of identity, for Quine, allow us to decide, in principle, whether  $a$  is the same as  $b$ , where " $a$ " and " $b$ " refer to objects of the kind in question.

This is essentially the Fregean notion of identity criterion stated by Frege before he observed that it does not allow us to decide identity sentences where one of the terms refers to any object. In other terms, an identity criterion, in Quine's perspective, is not required to discriminate an object of the kind of objects for which it is proposed from any other object of whatever

kind. Moreover it is very likely that identity among objects of the kind for which the criterion is proposed is not allowed to occur in the identity condition. So, for Quine, an identity criterion has to answer – in a non-circular way – to the (OQ) question and it does not answer the (OQ1) question.

As for Frege, it is plausible to assume that “to decide  $p$ ”, where  $p$  is an identity sentence, means that – on the basis of the relevant information –  $p$  is logically implied or  $\neg p$  is logically implied, but in this case it seems to us that the logical implication could be understood in the usual model-theoretic sense. The notion of relevant information raises at least all the problems connected with Frege’s analysis, but, as for Frege, let us consider the particular case in which all the relevant information is *a priori* available.

If such information is specified in the first order language and it contains numbers theory, on the basis of the semantic completeness of first order logic, decidability – in the sense specified above – implies deductive decidability, i.e. for each identity sentence  $p$ ,  $p$  is formally deducible or  $\neg p$  is formally deducible. So, since for a such a theory deductive deducibility implies recursive decidability, on the basis of Lombard’s argument any sentence is recursively decidable, against the existence of problems which are recursively undecidable. We should conclude that identity criteria cannot be such that they logically imply  $p$  or logically imply  $\neg p$ , for every identity sentence  $p$ . This conclusion follows when descriptive terms are allowed to occur in the identity sentences, where descriptive terms are terms built by means of the description operator or names introduced in connection with suitable conditions.

There are some other problems in Quine’s proposal which were pointed out by Kripke (in particular in [Kripke 1978]). He claims that such a case as the well-known problem of the Ship of Theseus gives us an example of a problem concerning ordinary physical objects not decidable on the basis of the relevant information. It seems to follow that for the ordinary notion of physical objects a criterion of identity, in the Quinean sense, is not available. However, Kripke argues that the existence of sentences which cannot in principle be decided does not delegitimize the ordinary notion of physical object. If this notion were delegitimized, the scientific notion of physical object would be analogously delegitimized by the existence of issues of identity concerning elementary particles which are not in principle decidable [Lowe 1994]. If we accept the above criticisms to the Quinean notion of identity criteria, we can conclude – at this stage – that identity criteria do not give us a method of decision for identity sentences. Moreover, doubts have arisen about the possibility that identity criteria provides an explicative, non-circular, introduction of the concept of the objects for which they are formulated.

## 6. Reduction and identity criteria. On Quine and Kripke again

According to Kripke, Quine takes identity criteria as principles reducing issues of identity of a given kind to relations between objects of a more basic kind. Such a conception is attributed to Quine after an analysis of the Quineian exposition of the identity criterion for rivers proposed by Quine in [Quine 1953]. There are two main criticisms proposed by Kripke to this kind of reductivistic conception of identity criteria.

First objection. If identity criteria have to provide an analysis of identity, we have to admit objects for which there are no reductive criteria of identity and from whom we move on in order to give identity criteria for less basic objects [Kripke 1978, p. 38]. Otherwise we run into an *infinite regress*. Consider this example of identity criterion:

(MO=) Materials objects are identical if and only if they occupy the same place at any time

One could ask a criterion of identity for the notion of *place* and this criterion has to be given in terms of entities different from those which the first ones are reducible to. It is not clear what these entities can ever be, but there should be some, if identity among places is to be reduced to identities among more basic entities. Then, keeping on applying the same kind of demand, one gets involved in an *infinite regress*. In order to stop this infinite regress, a reductivistic philosopher could introduce some *scientific standards* and suppose that a criterion of identity is adequate if and only if the right-hand side of the criterion is an ontological reduction of the left-hand side in terms of the selected scientific standard. This seems to us to be, more or less, for example, Sellars' answer. He argues that "chairs" do not really exist. There are objects that really exist and which answer to what the layman calls "chairs", but the objects called "chairs" by the layman are part of a pre-scientific, intuitive, picture of the world. Chairs really are... and here the reduction follows on the basis of the scientific standard adopted. Such a kind of explanation makes the whole question of the adequacy of identity criteria depend on the *reference standard* adopted. Now the problem is: which standard?

Second objection. It specifically concerns some notions considered more basic than others, such as, for example, the notion of *momentary-thing* or the notion of *time-slice*. Concerning the notion of *momentary-thing*: Kripke claims that such a notion is not understandable without presupposing the notion of an enduring object, and so the last one is not conceivable as a notion derived from the first one.

Finally one can address a more general criticism of the concept of reduction – a concept that seems to be implied in the quineian conception of



identity criteria – in agreement with some Kripkian remarks and with some of the thesis developed by Wiggins in [Wiggins 1980].

When identity conditions for certain objects are stated, it is usual to refer to other objects for which identity is presupposed. We claim that the understanding of such conditions presupposes also an idea for which the identity criterion is proposed. So, the formulation of an identity criterion presupposes at least one partial understanding of the objects for which the criterion is proposed.

## 7. Conclusions

Our main conclusion is that identity criteria do not provide ontological legitimacy. Moreover identity criteria are not explicative of identity and they do not give us a method of decision for identity sentences. Now, let us consider the problem stated in the introduction. We said that one way to stop the chain of queries generated by the first question “How many objects are there?” is to argue that this one does not hold a determinate meaning. It would be rightly raised if there was a specification of the kind or sort of objects to count. Doing so, we associate to the sortal term, in some way, an *identity criterion*. So, a question concerning counting is determinate, or appropriate, if and only if it is specified by a sortal and there is a *criterion of identity* for it, where the expression “criterion of identity” occurring in the above biconditional is a shorthand for “identity criterion in its standard meaning”. In the paper we characterise what we mean for “identity criterion in its standard meaning”. If so, then the solution proposed to stop the chain of queries started from the question: “How many objects are there?” is false.

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